Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Differential Equations

Subject Code: 5SC01DIE1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 14/03/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a. Determine the radius of convergence of $(1-x)^{-1}$. (02)

b. Find the value of $\int_{-1}^{1} P_3^2(x)\,dx$. State the result you use. (02)

c. Write generating function of Bessel's function. (01)

d. Write Rodrigue's formula. (01)

e. Bessel's function $J_n(x)$ is even function if n is odd. Determine whether the statement is True or False. (01)

Q-2 **Attempt all questions** **(14)**

a. Find the power series solution about $x=0$ of the equation (06)

$$y'' + xy' + y = 0, y(0) = 3, y'(0) = -7.$$

b. Using the method of variation of parameters, solve (05)

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}.$$

c. Show that $x=2$ is an irregular singular point of the equation (03)

$$2x(x-2)^2y'' + 3xy' + (x-2)y = 0.$$

OR

Q-2 **Attempt all questions** **(14)**

a. Find the power series solution near $x=0$ of the equation $(1-x^2)y'' - 2xy' + p(p+1)y = 0$, where p is an arbitrary constant. (06)

b. Using the method of variation of parameters find the general solution of the differential equation $(D^2 - 2D + 1)y = 3x^{\frac{3}{2}}e^x$. (05)

c. Define: Singular point, Regular singular point and irregular singular point. (03)

Q-3 **Attempt all questions** **(14)**

a. Use the method of Frobenius to find one solution near $x=0$ of $x(x-1)y'' + (3x-1)y' + y = 0$. (06)

b. Show that (05)

1) $J_{n+1}(x) = \frac{2n}{x}J_n(x) - J_{n-1}(x)$,
2) $J_n'(x) = J_{n-1}(x) - \frac{n}{x}J_n(x)$.

	c.	Express $2 - 3x + 4x^2$ in terms of Legendre's polynomials.	(03)

OR

Q-3		**Attempt all questions**	**(14)**
	a.	Determine the values of $J_{\frac{1}{2}}(x)$ and $J_{-\frac{1}{2}}(x)$.	(06)
	b.	Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x), \forall\, n \geq 2$.	(05)
	c.	Expand $f(x)$ in the form $\sum_{n=0}^{\infty} c_n P_n(x)$, where $f(x) = \begin{cases} 0, & -1 < x < 0 \\ x, & 0 < x < 1 \end{cases}$.	(03)

SECTION – II

Q-4		**Attempt the following questions**	**(07)**
	a.	Form the partial differential equation by eliminating the arbitrary constants a and b from $\log(az-1) = x + ay + b$.	(02)
	b.	Find a complete integral of $pqz = p^2(xq + p^2) + q^2(yp + q^2)$.	(02)
	c.	Define: Quasi-linear equation.	(01)
	d.	Write Gauss's hyper geometric equation.	(01)
	e.	Write Lagrange equation.	(01)

Q-5		**Attempt all questions**	**(14)**
	a.	Find a complete integral of $p^2z + q^2 - 4 = 0$ by Jacobi's method.	(06)
	b.	Prove that if $\vec{X}$ is a vector such that $\vec{X} \cdot curl\, \vec{X} = 0$ and μ is an arbitrary function of x, y, z, then $\mu\vec{X} \cdot curl\, \mu\vec{X} = 0$.	(05)
	c.	Find a complete integral of $p - e^q = 0$.	(03)

OR

Q-5		**Attempt all questions**	**(14)**
	a.	Show that the Pfaffian differential equation $$yzdx + 2xzdy - 3xydz = 0$$ isintegrable and find its solution.	(06)
	b.	Prove that a necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$ a relation $F(u,v) = 0$, not involving x and y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$.	(05)
	c.	Find a complete integral of $z^2p^2 + q^2 - p^2q = 0$.	(03)

Q-6		**Attempt all questions**	**(14)**
	a.	Prove that 1) $(1-x)^n = F(-n, 1; 2; -x)$, 2) $\lim_{\beta \to \infty} F\left(\alpha, \beta; \gamma; \frac{x}{\beta}\right) = F(\alpha, \gamma; x)$, 3) $\frac{d}{dx}[F(\alpha, \beta;\ \gamma;\ x)] = \frac{\alpha\beta}{\gamma}\ F(\alpha+1, \beta+1;\ \gamma+1;\ x)$.	(06)
	b.	Using Picard's method of successive approximation, find a sequence of two functions which approach solution of the initial value problem, $\frac{dy}{dx} = e^x + y^2, y(0) = 1$.	(05)
	c.	Eliminate arbitrary function f from $z = e^y f(x+y)$.	(03)

OR

Q-6		**Attempt all questions**	**(14)**

a.	Show that the equations $f(x,y,p,q)=0$ and $g(x,y,p,q)=0$ are compatible if $\frac{\partial(f,g)}{\partial(x,p)}+\frac{\partial(f,g)}{\partial(y,q)}=0$. Verify that the equations $p=P(x,y)$ and $q=Q(x,y)$ are compatible if $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$.	(06)
b.	Solve: $y^2p-xyq=x(z-2y)$.	(05)
c.	Form a partial differential equation by eliminate arbitrary function f from $z=f\left(\frac{y}{x}\right)$.	(03)